Gröbner Basis and Planar Graph

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Abstract. Using the Gröbner basis of an ideal generated by a family of polynomials we prove that every planar graph is 4-colorable.

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We are given a graph G with n vertices. Between two any vertices there is at most one edge. We want to color the vertices by 4 colors such that no two vertices connected by an edge are colored the same way. We let $\zeta = i \in \mathbb{C}$ be the 4th primitive root of unity $(i^4 = 1)$. Let $x_1, x_2, \cdots, x_n$ be variables representing the distinct vertices of the graph G. Each vertex is to be assigned one of colors $1, i, i^2, i^3$. This means that $x_k^4 - 1 = 0$ for $k = 1, 2, \cdots, n$. Moreover, if x_j and x_k are connected then $x_j^3 + x_j^2 x_k + x_j x_k^2 + x_k^3$. Let I be the ideal generated by $x_k^4 - 1$ for $k = 1, 2, \cdots, n$ and $x_j^3 + x_j^2 x_k + x_j x_k^2 + x_k^3$ if x_j and x_k are connected. We should prove that $I \neq \mathbb{C}[z_1, z_2, \cdots, z_n]$.

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References

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